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NEW CORRELATIONS AND TABLES OF THE COEFFICIENT OF VISCOSITY OF WATER AND STEAM UP TO 1000 BAR AND 1000°C

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Abstract—New correlations and tables of the coefficient of viscosity of water and steam are presented. The correlations have the same general format as those appended to the International Skeleton Tables (800 bar, 700°C) but take account of recent work, thus permitting values up to the higher limits of 1000 bar and 1000°C to be derived. The paper gives references to all the leading works and recommends that systematic and extensive measurements be made in the temperature range 0–400°C at all pressures.

NOMENCLATURE

p, pressure [bar];
p_s, saturation pressure [bar];
T, temperature [°K], $T = t + 273{\cdot}15$;
t, temperature [°C];
A, B, C, constants;
a, b, c, d, e, f, constants.

Greek symbols

η, dynamic viscosity [poise—g/cm s];
η_1, dynamic viscosity at 1 bar pressure;
η_s, dynamic viscosity at saturation;
η_{pT}, dynamic viscosity at pressure p and temperature T;
θ_0, inversion temperature [°K];
ρ, density [g/cm³];
α, β, γ, constants.

INTRODUCTION

THE PUBLICATION [1] of Skeleton Tables of Viscosity and Thermal Conductivity by the Sixth International Conference on the Properties of Steam shows the Conference in a new role. The main interest had in the past been the Thermodynamic Properties and, no doubt, these will remain a major interest in the future, but now for the first time the Transport Properties have been recognized. Further, associated with the first Skeleton Tables of Viscosity and Thermal Conductivity are formulae suitable for interpolation in the Tables. The presentation of formulae marks a new departure in the presentation of agreed values, a departure which may have wider implications when extended to the Thermodynamic Properties.

The process of preparing Skeleton Tables is of necessity a long process and it is perhaps inevitable that the results of some of the latest researches fail to be considered, although it is to be expected that all observations subsequent to the publication of a Skeleton Table would fall within the tolerances of this Table. The purpose of this paper is to present some new correlations for viscosity, which take account of the extensive observations of two of us (B.L. and A.K.R.) at the University of Glasgow and to indicate what further experimental work is required. The results of a group of Japanese workers [2] are also compared with the values obtained in this paper.

SOURCES

The sources of experimental work, on which

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the correlations are based, have been taken from surveys in the published work of Kjelland-Fosterud [3], Whitelaw [4], Ray [5] and Latto [6], each of whom has reviewed all the more important publications. Comprehensive surveys [7–9] have also been made, although the primary references are discussed in the published papers. These primary references, together with the work of Dorsey [10] who has appraised all the experimental work carried out prior to 1940, form an adequate basis on which to form the correlations.

The liquid phase ($t \ngtr 375°C$)

At atmospheric pressure and in the temperature range 0–40°C the coefficient of viscosity appears to be well defined, as may be seen from the entries in Table 1. This table gives values of the ratio η_t/η_{20} and shows the recommended values, according to various authorities, at different times over the past 50 years. It is seen that the value of the coefficient at any temperature is dependent on the reference value for the coefficient at 20°C. The most precise measurement of the coefficient of viscosity is believed to be that of Swindells, Coe and Godfrey [22] who reported a value of 1·0019 centipoise $\pm 0·0003$ centipoise at 20°C, using a capillary viscometer. This determination is supported by Roscoe and Bainbridge [23] who state a value of 1·0025 centipoise $\pm 0·0005$ centipoise at 20°C, a value obtained using an oscillating vessel technique. The close agreement of these two determinations, using entirely different apparatus, leads to the conclusion that a reference value for η_{20} = 1·002 centipoise could safely be adopted, subject to an uncertainty not exceeding 0·001 centipoise.

Acceptance of 1·002 centipoise as the value of η_{20} enables a comparison to be made between Weber's recommendation (column 11, Table 1) and Dorsey's recommended values of 1940 [10]. At the same time the corresponding values obtained in this work can be included along with those given by the correlating formula accompanying the 1964 Skeleton Table (Appendix C, 2.4, of reference [1]). This intercomparison is shown in Table 2. It is to be noted that the recommended values of Dorsey and Weber agree to within 0·3 per cent, whereas the correlating formula of the Skeleton Table gives values which are almost the same as those obtained in

Table 1. Ratio of η_t/η_{20} for liquid water

Col.	1	2	3	4	5	6	7	8	9	10	11
Year	1911	1918	1929	1933	1942	1943	1944	1949	1952	1952	1955
Temp. (°C)											
0	1·7864	1·7832	1·7784	—	—	1·7829	—	—	1·7668	1·7897	1·7885
5	1·5159	1·5112	1·5056	—	—	1·5140	—	1·5154	1·5079	1·5178	1·5170
10	1.2932	1·3012	1·2984	—	—	1·3035	—	—	1·3014	1·3044	1·3043
15	1·1312	1·1347	1·1348	—	—	1·1358	—	—	1·1353	1·1360	1·1360
20	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	—	1·0000	1·0000	1·0000	1·0000
25	0·8892	0·8893	0·8872	0·8885	0·8885	0·8885	0·8885	—	0·8886	0·8885	0·8885
30	0·7948	0·7967	0·7933	0·7947	—	0·7959	0·7960	—	0·7958	0·7959	0·7959
35	0·7184	—	—	—	—	—	—	—	0·7177	0·7175	0·7179
40	0·6522	0·6527	0·6480	0·6519	0·6513	0·6518	0·6518	0·6518	0·6514	0·6517	0·6518
50	—	0·5457	0·5445	—	0·5456	0·5460	—	—	—	—	—
60	—	0·4665	0·4658	0·4660	0·4657	0·4659	—	—	—	—	—

1. Bingham and White [11]
2. Bingham (correlation) [12]
3. Dorsey (correlation) [13]
4. Geddes [14]
5. Swindells [15]
6. Cragie (correlation) [16]
7. Coe and Godfrey [17]
8. Hardy and Cottington [18]
9. Kampmeyer [19]
10. PTB 1952 [20]
11. Weber [21]

Table 2

Temp. (°C)	η (centipoise)			
	Dorsey [10]	This work	S.T., 1964 [1]	Weber [21]
0	1·794	1·757	1·753	1·792
10	1·310	1·301	1·300	1·307
20	1·009	1·002	1·002	1·002
30	0·8004	0·797	0·797	0·798
40	0·6536	0·651	0·651	0·653
50	0·549	0·544	0·554	—
60	0·470	0·462	0·463	—
70	0·407	0·400	0·400	—
80	0·357	0·350	0·351	—
90	0·317	0·311	0·311	—
100	0·284	0·278	0·279	—

this work, yet both differ from Dorsey and Weber at temperatures below 20°C. At this point it is necessary to note that the correlating formula of the Skeleton Table has a tolerance attributed to it of $\pm 2{\cdot}5$ per cent and, since the formula used in this work has a similar form, the same figure could likewise be applied here.

It may be concluded that the correlation of Dorsey [10] is supported over the range 0–40°C by the recent measurements of Weber, and that the value of the coefficient of viscosity at 20°C is 1·002 centipoise. The discrepancies between the values of Dorsey and Weber below 20°C and the values given by the correlating formulae of the Skeleton Table and of this work are due to the attempts to take account of observations at pressures greater than one atmosphere.

Prior to 1940 very little experimental work on the effect of pressure on liquid viscosity had been carried out, the leading experiments being those of Bridgman [24], de Haas [25], Sachs [26], Warburg and Sachs [27] and Sigwart [28]. The observations of Sachs and of Warburg and Sachs do not agree with recent measurements and must be disregarded. Those of Bridgman were obtained, in the main, at pressures in excess of 1000 atm and since this pressure is the upper limit considered in this paper no account will be taken of Bridgman's measurements, although these, it is believed, are likewise not in accord with recent determinations. The observations of de Haas, with respect to the saturated liquid at 124·0°, 142·2° and 153°C are in accordance both with the correlation given here and with that of the Skeleton Table. The high pressure measurements of Sigwart do not fit the pattern of recent experiments. The ranges of temperature and pressure of Sigwart's experiments were 275–380°C and 25–270 atm, ranges including a region where a very high degree of stability in the apparatus is required. The scatter of Sigwart's results and the difficulties which he experienced with his ring balance show that this stability was not achieved. Subsequent to 1940 the experimental work of Timrot and Khlopkina [29], Schmidt and Mayinger [30], Moszynski [31] and Tanaka [2] has extended greatly the number of observations in the liquid phase. All these workers have made measurements in the vapour phase, using the same apparatus, and for this reason greater weight must be given to their liquid observations, since the vapour results agree so well, than to those observations obtained prior to 1940 which covered relatively small ranges of temperature and pressure.

Summary. Sources used directly in the correlation given here:

Swindells, Coe and Godfrey [22]
Timrot and Khlopkina [29]
Schmidt and Mayinger [30]
Moszynski [31].

Table 3. Viscosity of "saturated" liquid water

$$\eta_e = 239{\cdot}40 \times 10^{248{\cdot}37/(T-140)}$$

	Temperature °C	Viscosity (measured) (micropoise)	Viscosity (equation 2) (micropoise)	$\frac{\eta - \eta_e}{\eta_e} \times 100\%$
Weber [21]	0	17919	17569	1·99
	5	15199	15029	1·13
	10	13068	13011	0·44
	15	11382	11365	0·15
	20	10020	10020	0
	25	8902	8903	0
	30	7974	7969	0·06
	35	7193	7181	0·11
	40	6531	6509	0·34
Moszynski [31]	24·03	9093	9104	−0·12
	28·25	8256	8278	−0·27
	40·86	6391	6403	−0·18
	73·22	3839	3821	0·47
	103·77	2711	2676	1·31
	186·52	1483	1440	2·99
de Haas [25]	124·0	2232	2213	0·86
	142·2	1925	1910	0·79
	153·0	1805	1767	2·15
Extrapolated data	17·5	10510	10590	1·12
	50	5480	5465	−0·55
	100	2910	2810	−0·97
	200	1400	1368	−2·63
	250	1200	1075	−0·94
	300	933	900	−0·45
	300	917	905	2·16
	194	1387	1361	−0·07
	83	3422	3410	−0·24

Sources used *indirectly* in the correlation given here:

Tanaka [2]
Weber [21].

The vapour phase ($t \nless 375$°C at supercritical pressures)

The region defined here as the vapour phase may best be divided into three sub-regions, (a) low pressure (b) high pressure and (c) sub-critical pressure. The low pressure region is for the purposes of this chapter the one atmosphere line.

(a) *Low pressure.* One of us (B.L.) has recently reported [6, 9] viscosity values for superheated steam at one atmosphere pressure and, at the same time, has reviewed all the other leading works among which may be included those of Bonilla [32], Kestin [33] and Shifrin [34]. The most precise measurements at low temperatures would seem to be those of Kestin. However, only a small range of temperature, barely exceeding 300°C, is covered. The agreement between the experimental work of Kestin, Shifrin and Latto is good, but the results of Bonilla over the greater part of the temperature range are some four per cent lower. Comparison between these four experimenters is shown in Fig. 1 using as the basis for comparison the correlating equation (4) adopted in this paper. It is clear that Bonilla's

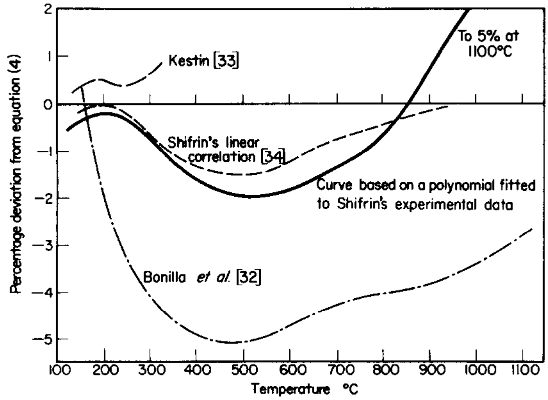

FIG. 1. Deviation of Shifrin's, Bonilla's and Kestin's recommended values, for the 1 atm line, from equation (4).

observations must be set aside and that only the results of Shifrin and Latto can be used over a wide temperature range. Figures 2 and 3 show the deviations of the experimental points obtained by Shifrin and Latto from equation (4) given in this paper. In Fig. 2 the circles represent Shifrin's actual experimental measurements and in Fig. 3 the various symbols represent Latto's experimental results, each symbol referring to an experimental point obtained with the same capillary. It can be seen that there is good agreement between the data of Shifrin and Latto although Shifrin has only 14 experimental points above a temperature of 600°C which compares rather unfavourably with the large number of measurements obtained by Latto, as can be seen from Fig. 3.

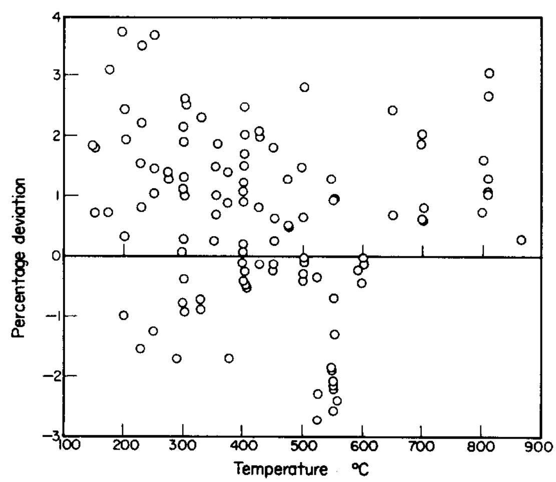

FIG. 2. Deviation of Shifrin's experimental results from the recommended values (equation 4).

The main question to decide is whether to combine the observations of Latto, Shifrin and Kestin or keep them separate. There is a natural preference for adhering to the worker whose observations cover the whole temperature range and not to include the observations of the other experimenters. This is to some extent justified by the fact that the agreement between these workers is so close. From a separate analysis of Shifrin's experimental data, which he himself fitted by a linear equation in temperature, it was found that his data were, in fact, better fitted by a polynomial and this curve is shown as a full line in Fig. 1, along with the broken line which represents Shifrin's own correlation. It is clear that, using Shifrin's data alone, extrapolation of the polynomial to temperatures above that at which Shifrin has an experimental point could lead to values some five per cent higher than those which were obtained experimentally by Latto at temperatures near to 1100°C. In view of the close agreement between Shifrin and Latto over the range 100–800°C it was decided to use only Latto's equation since this also represented the only data available at temperatures in excess of 860°C.

It is to be noted that the tolerance on the viscosity of superheated steam at one bar pressure in the temperature range 100–300°C is ± 1 per cent and from 300–700°C is ± 3 per cent, according to the correlating formula of the 1964 Skeleton Table. These tolerances are more than acequate to include within their bounds the experimental measurements of Kestin, Shifrin and Latto, and until further experimental work is carried out, in which the precision of measurement is so improved that it is possible to distinguish between one experimenter and another, the correlation proposed here would seem to be the best of the possible combinations of data.

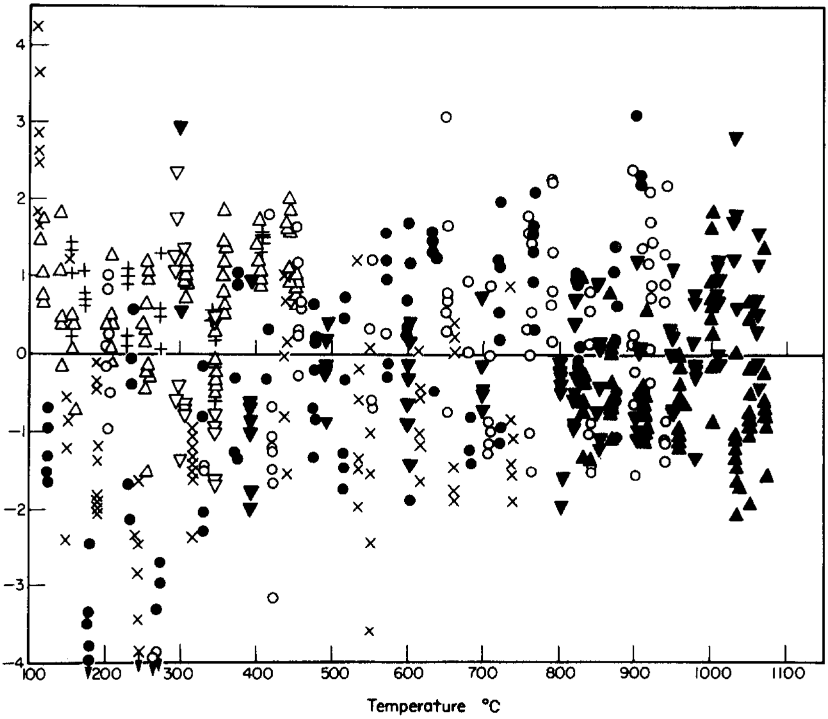

FIG. 3. Deviation of Latto's experimental results from the recommended values (equation 4).

Summary. Sources used directly in the correlation given here:

Latto [6, 9].

Sources used indirectly in the correlation given here:

Shifrin [34]
Kestin [33].

(b) *High pressure.* This region may be considered as being bounded by the 1 bar and 1000 bar isobars together with the 375°C and 1000°C isotherms, a region in which many of the observations are at supercritical pressures. Experimental results have been reported by Shugaiev [35], Timrot [36], Timrot and Khlopkina [29], Hawkins [37], Jackson [38], Osborne [39], Kjelland-Fosterud [3], Schmidt and Mayinger [30], Whitelaw [4], Ray [5], Barnett, Jackson and Whitesides [40] and Tanaka [2]. The observations of Shugaiev will not be considered since their reliability is in question and the later work of Timrot and of Timrot and Khlopkina may be considered to replace the earlier work of Shugaiev. The experiments of Jackson and of Barnett, Jackson and Whitesides with an annular viscometer are not in agreement with the large number of results obtained from capillary type apparatus by a number of other workers. In view of the reliability of the capillary method it is considered that the results obtained with the annular viscometer must be inferior, although the reason for the discrepancy poses some problems. The measurements of Osborne include the isotherms of 450°F (232°C), 500°F (260°C) and 900°F (482°C) and lie mainly outside the lower temperature limit of 375°C being considered. More recently Witzell [41] has used the same apparatus but results are only given up to 316°C. The measurements of Kjelland-Fosterud were revised by Whitelaw and this revision may be considered to be contained within the measurements of Whitelaw. This leaves only the work of Timrot, Timrot and Khlopkina, Whitelaw, Ray, Schmidt and Mayinger, and Tanaka as the leading experimenters in this field.

Summary. Sources used directly in the correlation given here:

Timrot [36]
Timrot and Khlopkina [29]
Whitelaw [4]
Ray [5]
Schmidt and Mayinger [30].

Sources used *indirectly* in the correlation given here:

Tanaka [2].

(c) *Sub-critical pressure.* The most precise determinations are considered to be those of Kestin and Richardson [33] although there are observations by other workers notably Witzell [41], Shugaiev [35], Timrot [36], Schiller [42] and Sigwart [28] among others. In this region the pressure effect ($\partial\eta/\partial p$) is negative and only the measurements of Kestin and Richardson are sufficiently precise to show this effect. Tanaka's calculations would seem to support the observations of Kestin and Richardson and it is believed that confirmatory experiments are in progress in the U.S.S.R. The only unsatisfactory aspect is the absence of a single set of precise measurements covering the low density region, where the pressure effect is negative, and the region of higher densities, where the effect is positive. Consequently there is a region, somewhat poorly defined between 300° and 375°C, a region which can only be bridged by graphical means.

Summary. Source used directly in the correlation given here:

Kestin and Richardson [33].

CORRELATIONS*

The liquid phase, 0–300°C *and from saturation to* 1000 *bar pressure*

The range of the actual experimental points

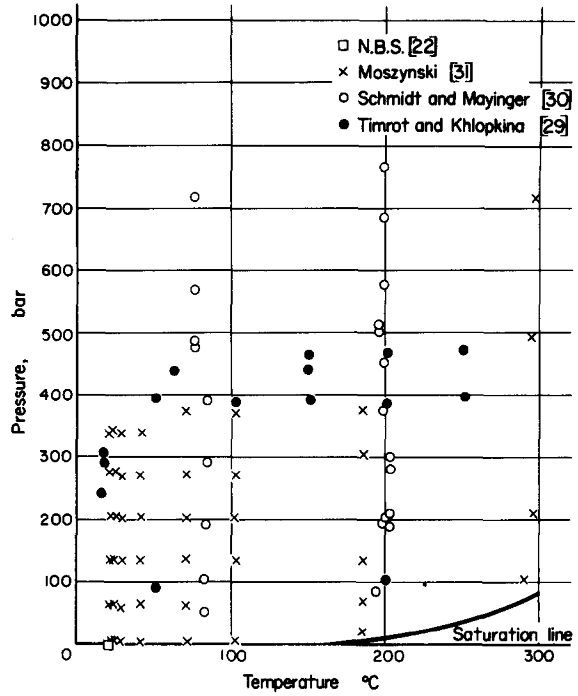

FIG. 4. Experimental points used for the compressed water correlation (equation 1).

from references [22, 29–31], already discussed above, is shown in Fig. 4, where the ordinate represents pressure and the abscissa temperature. From Fig. 4 it might appear that the field is adequately covered by observations except at the high pressures. However, when one examines the magnitude of the property itself and, in particular, its rate of change from 0° to 100°C and in the region of 300°C, it is clear that the raw data provide a weak basis on which to form the correlation.

The systematic series of measurements of Moszynski show the existence of an inversion temperature, which appears to lie between 30° and 40°C at low pressures (< 400 bar). The inversion temperature is that temperature for liquid water above which $\partial\eta/\partial p$ is positive and below which $\partial\eta/\partial p$ is negative. Moszynski recommends a value for the inversion temperature (θ_0) of 35°C (308°K) although analysis of

* Equations (1)–(7) below give values of viscosity in micropoise.

the experimental work yields a higher value, 37°C (311°K). This latter value is adopted here and this inversion temperature is assumed to be the same at all pressures up to 1000 bar, the limit of the correlation given here.

While Moszynski's measurements cover an important part of the field in a systematic way, his value for the viscosity at 20°C is approximately one per cent lower than the value of Swindells, Coe and Godfrey (the N.B.S. value) [22]. Since the N.B.S. value has been accepted as $\eta_{20} = 10020$ micropoise in the preceding appraisal with a reasonable degree of confidence it was considered that for the purposes of the correlation all Moszynski's observations should be multiplied by 1·01 so as to bring his measurement in the vicinity of 20°C into close agreement with the N.B.S. value. By applying this weighting factor to Moszynski's observations it is possible to correlate all the data of references [29–31] and reproduce the value of η_{20} as 10020.

The correlation equation is of the form

$$\eta_{pT} = \eta_s \times F \tag{1}$$

where η_{pT} is the dynamic viscosity at pressure p bar and temperature T°K and η_s represents the viscosity along the saturation line. The quantity η_s is given by

$$\eta_s = \alpha 10^{\beta/(T-140)} \tag{2}$$

Equation (2) has a semi-theoretical basis and none of the alternative equations investigated proved as satisfactory.

The factor, F, is a pressure correction factor and may be written in the form

$$F = 1 + \left[\frac{(p - p_s)(T - \theta_0)}{10^6}\right] \{a + b(T - \theta_0) + c(T - \theta_0)^2\}. \tag{3}$$

The values of α, β and γ were derived from previous work by one of us (E.A.B. [43]) and the value of θ_0 was taken as 311°K, as discussed above. The constants a, b and c were determined by the method of least squares and found to be

$$a = 1{\cdot}0049 \qquad \alpha = 239{\cdot}4$$
$$b = 2{\cdot}6016 \times 10^{-4} \qquad \beta = 248{\cdot}37$$
$$c = -1{\cdot}0323 \times 10^{-6} \qquad \gamma = 140°\text{K}$$
$$\theta_0 = 311°\text{K}$$

The results of the analysis are shown as a deviation plot in Fig. 5 and it is believed that the correlation gives slightly better agreement with the experimental data, even allowing for the weighting of Moszynski's data, than does the corresponding equation which appears in the appendix to the Skeleton Table.

Although the measurements of Tanaka have not been used directly in the correlation for compressed water, Table 4 shows there is close agreement between the 1964 Skeleton Table value, that given here and Tanaka's correlated value. In fact Tanaka's table entry is only just outside the Skeleton Table tolerance at pressures in excess of 700 bar and at temperatures below and including 50°C. Unfortunately neither Tanaka nor any other worker has experimental points in the range 0–50°C at pressures above 700 bar and there is no way of resolving discrepancies between various equations whose forms in any case clearly leave much to be desired.

The vapour phase

(a) *Low pressure, superheated steam* 100–1000°C *at* 1 *bar pressure.* Latto's correlating equation [6, 9] is of the form

$$\eta_1 = a + bT + cT^2 + dT^3 + eT^4 + fT^5 \tag{4}$$

where η_1 is the dynamic viscosity at a pressure of 1 bar

and where $a = -0{\cdot}65634 \times 10$

$b = 0{\cdot}26700$

$c = 0{\cdot}25500 \times 10^{-3}$

$d = -0{\cdot}13303 \times 10^{-6}$

$e = -0{\cdot}22475 \times 10^{-10}$

and $f = 0{\cdot}18488 \times 10^{-13}$.

This equation was the best fit to the data as compared with lower and higher polynomials.

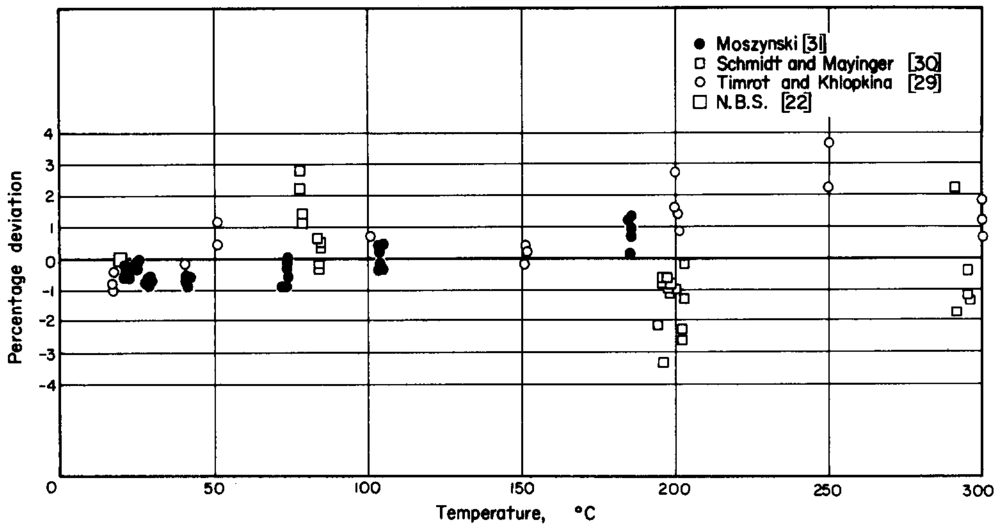

FIG. 5. Deviations of experimental observations for compressed water from correlating equation (1).

Table 4. Comparative data for viscosity (micropoise) of water and steam

bar		0°C	50°C	300°C	375°C	700°C	1000°C
	a	17500 (400*)	5440 (140)	202·5 (2·0)	233 (7)	365 (11)	—
1	b	17557	5435	204	236	368	475
	c	—	5452	202·5	233·1	365·4	—
	a	17400 (400)	5470 (140)	943 (24)	597 (24)	389 (16)	—
250	b	17392	5452	935	592	389	490
	c	—	5549	930·8	601·8	385·7	—
	a	17200 (700)	5490 (220)	1010 (40)	762 (30)	423 (17)	—
500	b	17227	5469	994	760	421	509
	c	—	5647	985·1	793·8	417·8	—
	a	17100 (700)	5520 (220)	1080 (40)	846 (34)	478 (19)	—
800	b	17030	5489	1065	844	475	535
	c	—	5765	1050	855·4	474·6	—
	a	—	—	—	—	—	—
1000	b	16898	5495	1112	885	518	553
	c	—	5843	1094	896·2	521·2	—

(a) S.T., 1964 [1] (800 bar; 700°C). (b) Present paper. (c) Tanaka [2].
* Tolerance.

Equation (4) is valid over the temperature range 100–1100°C and the standard deviation of the experimental points from this equation is $\pm 1{\cdot}22$ per cent for an assembly of over 500 separate observations.

(b) *High pressure,* 375–1000°C *and* 1–1000 *bar.* In this region it is found that a correlation may be reached on the basis of the equation

$$\Delta\eta = \eta - \eta_1 = A\rho + B\rho^2 + C\rho^3 \qquad (5)$$

where $\Delta\eta$ is the viscosity "excess" and ρ is the density in g/cm^3. This equation was put forward at the Meeting of the International Coordinating Committee on the Properties of Steam at

Table 5

Run No.	Values of constants after each run A	B	C	Points rejected after each run
1	173·19	1859·55	−1619·17	179, 181, 182, 183, 185, 186, 187
2	267·77	1092·26	−393·51	76, 90, 170, 176, 177, 178, 180, 184
3	316·24	700·38	145·39	167, 168, 175, 206
4	313·60	736·69	92·01	169, 208
5	310·68	756·73	66·29	116
6	307·42	779·85	37·51	—

Munich (July 1962) and is given in the Appendix to the 1964 Skeleton Table. Until the precision and reproducibility of the measurements is improved this equation adequately represents the viscosity "excess" in this region.

The values of η_1 were taken from equation (4) and the density values from the V.D.I. [44] and Vukalovich [45] Steam Tables. The values of density had been determined from these sources on earlier occasions and since the densities agreed closely with the N.E.L. Steam Tables [46] it was decided to use the old values. It has to be borne in mind that there is almost one order of magnitude of difference between the precision with which viscosities are known and that with which densities are known. Consequently, the sources of the density values are not critical provided the values are in reasonably close agreement with one another.

The method of computation followed was similar to that used by Bruges [43] and Ray [5]. All the experimental data of references [39, 29, 44, 45 and 30] were subjected to a "least squares" analysis to give the coefficients A, B and C in equation (5). At this stage all points with deviations greater than three times the standard deviation (σ) from the equation value were rejected and the analysis repeated stage by stage until all the points lay within a scatter band of $\pm 3\sigma$. Five runs were necessary to achieve this object and the results are shown in Table 5, the reference numbers of the points agreeing with the numbers given by Bruges [43] to the original data. The pressure and temperature of each rejected point is given in Table 6 together with the source. The rejection of 22 points out of a total assembly of 271 reduces the standard deviation from $\pm 10{\cdot}74$ per cent to 2·38 per cent. The deviations of the experimental points, excluding rejected points, from the equation

$$\eta = \eta_1 + 307{\cdot}42\rho + 779{\cdot}85\rho^2 + 37{\cdot}51\rho^3 \tag{6}$$

are shown in Figs. 6 and 7.

The results of Tanaka are in good agreement with this correlation and Fig. 8 shows the percentage deviation of Tanaka's observations from equation (6). The plot is especially satisfactory

Table 6

Reference point	Pressure (kgf/cm²)	Temperature (°C)	Source
167	250	382·9	Whitelaw [4]
168	250	382·7	
169	250	383·6	
170	700	388·0	
175	700	487·0	
90	198·7	453·0	Timrot [36]
176	203	357·0	
177	209	362·7	
178	149·5	530·0	
179	254	382·0	
180	301	384·0	
181	351	385·5	
182	296·5	600·5	
183	254	600·9	
76	102	600	Timrot and Khlopkina [29]
186	28·7	650	
187	31·1	700	
116	315·5	391·5	Schmidt and Mayinger [30]
184	322	392·0	
185	620	595·0	
206	350	420·0	Ray [5]
208	350	468·2	

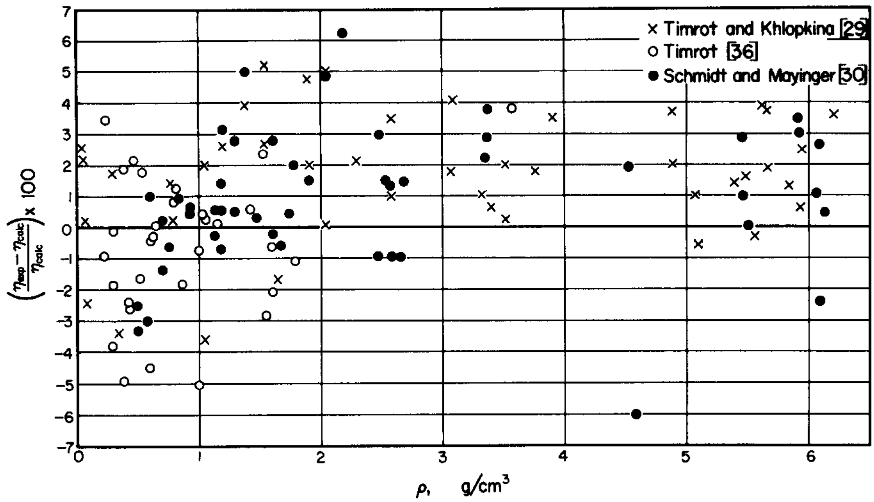

FIG. 6. Viscosity of steam—Deviation of experimental points from equation (6).

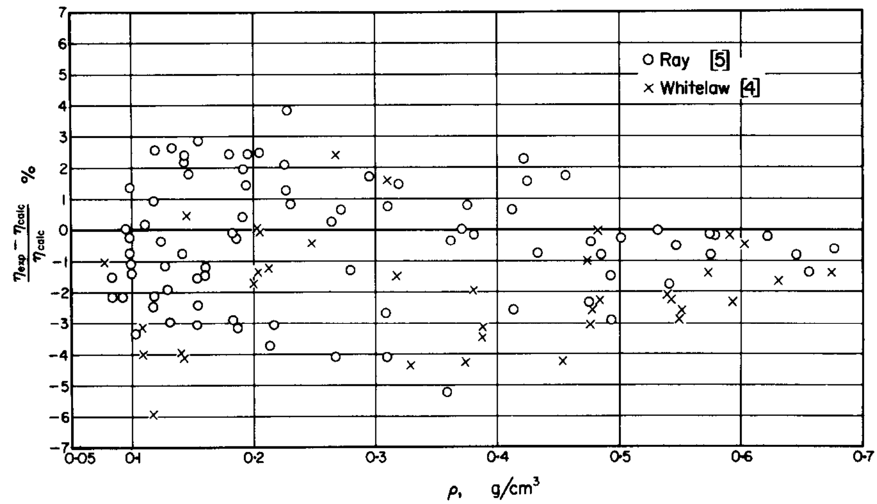

FIG. 7. Viscosity of steam—Deviation of experimental points from equation (6).

since only seven points lie outside the limits of ± 4 per cent, a figure representing the tolerance in this region of the Skeleton Table. A comparison between the various tables is set forth in Table 4, where it is seen that agreement with the entries from the three different sources is very good.

(c) *Sub-critical pressure*, 100–300°C *and from* 1 *bar to saturation pressure*. In this region the correlation of Kestin and Richardson [33] is accepted and used here. It is

$$\eta - \eta_1 = -\rho[1858 - 5{\cdot}90t] \text{ micropoise} \quad (7)$$

where ρ is the density in g/cm³ and t is the temperature in °C.

Construction of Tables 7 *and* 8

Entries in Table 7, given by equation (6), were obtained using density values in accord with the *N.E.L. Steam Tables* [46] and by extrapolation of these tables to give densities at temperatures above 800°C. These densities were carefully compared with those which would have been given by the older sources [44, 45] and the

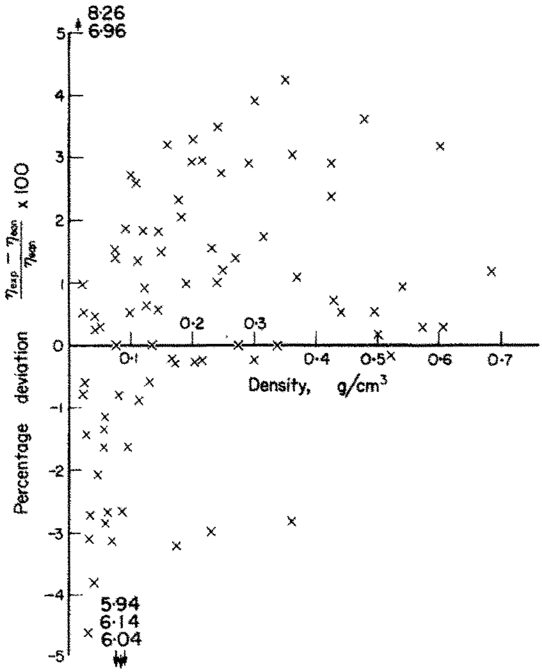

FIG. 8. Percentage deviation of Tanaka's [2] observations from equation (6).

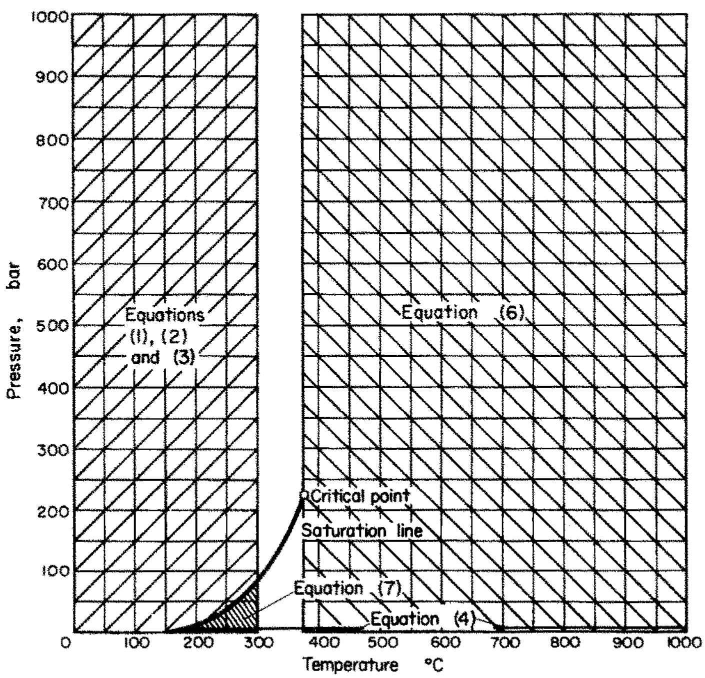

FIG. 9. Ranges of pressure and temperature covered by correlating equations.

Table 7. Viscosity (micropoise) of water and steam

bar	0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700	750	800	850	900	950	1000°C
1	17557	5435	121	142	162	183	204	225	236	246	257	267	278	288	308	329	349	368	388	406	424	441	458	475
5	17554	5436	2783	1804	160	182	204	225	236	247	257	268	278	288	309	329	349	368	388	406	424	442	459	475
10	17551	5436	2784	1805	159	181	204	225	237	247	258	268	278	289	309	330	349	369	388	406	424	442	459	476
25	17541	5437	2787	1808	1335	179	203	226	238	249	259	269	280	290	311	331	350	370	389	407	425	443	460	476
50	17525	5439	2791	1814	1340	1068	202	228	241	252	262	272	282	293	313	333	352	372	391	409	427	444	461	478
75	17508	5440	2795	1819	1346	1073	201	233	245	255	265	275	285	295	315	335	354	374	392	411	428	446	463	479
100	17492	5442	2800	1824	1351	1079	900	237	249	259	269	278	288	298	318	337	357	376	394	412	430	447	464	481
125	17475	5444	2804	1829	1357	1085	906	244	255	263	273	282	291	301	320	340	359	378	396	414	432	449	466	482
150	17459	5445	2809	1834	1362	1090	912	255	262	269	277	286	295	304	323	342	361	380	398	416	434	451	467	483
175	17492	5447	2813	1839	1368	1096	917	728	272	276	283	291	299	308	327	345	364	382	400	418	435	452	469	485
200	17425	5449	2817	1845	1373	1102	923	729	289	285	289	296	304	312	330	348	366	384	402	420	437	454	470	486
225	17409	5450	2822	1850	1379	1107	929	732	485	298	298	302	309	317	334	351	369	387	405	422	439	456	472	488
250	17392	5452	2826	1855	1384	1113	935	741	592	319	308	310	315	322	337	354	372	389	407	424	441	458	474	490
275	17376	5454	2830	1860	1390	1119	941	749	627	364	322	319	322	327	342	358	375	392	409	426	443	460	476	491
300	17359	5455	2835	1865	1395	1125	947	758	651	455	343	330	330	334	346	362	378	395	412	429	445	462	478	493
350	17326	5459	2844	1876	1406	1136	959	780	689	570	413	361	350	349	357	370	385	401	417	433	449	465	481	496
400	17293	5462	2852	1886	1417	1148	970	802	717	625	501	409	377	368	369	379	392	407	423	438	454	470	485	500
450	17260	5465	2861	1896	1428	1159	982	816	740	662	563	466	412	391	383	389	400	419	428	444	458	473	489	504
500	17227	5469	2870	1907	1439	1170	944	830	760	690	607	519	453	419	399	400	409	421	435	449	464	479	494	509
550	17194	5472	2879	1917	1450	1182	1006	845	777	714	641	562	494	451	417	413	419	429	441	455	469	483	498	513
600	17161	5475	2888	1927	1461	1193	1018	860	793	734	668	598	532	483	437	426	429	437	448	461	475	489	503	518
650	17128	5479	2896	1938	1472	1205	1029	870	807	752	691	627	565	514	458	441	440	446	456	467	480	493	508	522
700	17095	5482	2905	1948	1483	1216	1041	880	820	768	712	652	594	543	480	456	451	455	463	474	486	499	513	527
750	17062	5485	2914	1958	1494	1228	1053	892	833	783	730	674	619	570	502	472	463	465	471	481	492	504	517	530
800	17030	5489	2923	1969	1505	1239	1065	904	844	797	746	694	642	594	524	489	476	475	480	488	498	510	522	535
850	16996	5492	2932	1977	1516	1251	1076	916	855	809	761	712	659	605	545	506	489	485	488	495	504	515	527	539
900	16962	5494	2941	1983	1527	1262	1088	928	865	821	775	728	681	637	565	523	502	496	497	505	511	521	533	545
950	16929	5498	2950	1998	1538	1273	1100	939	875	833	787	743	697	656	584	539	516	507	506	511	517	526	537	549
1000	16898	5495	2959	2008	1549	1285	1112	950	885	843	800	757	714	673	603	555	529	518	516	519	524	532	542	553

Table 8. Coefficient of viscosity of water—Saturation values (micropoise)

Temperature (°C)	Liquid	Vapour
0	17570	88
10	13010	92
20	10020	96
30	7969	99
40	6509	102
50	5435	105
60	4624	108
70	3996	112
80	3502	115
90	3105	118
100	2782	121
110	2510	125
120	2290	128
130	2130	132
140	1940	135
150	1800	139
160	1680	143
170	1580	146
180	1490	150
190	1410	153
200	1330	157
210	1270	160
220	1210	164
230	1160	167
240	1110	171
250	1070	175
260	1030	179
270	989	184
280	953	189
290	925	194
300	896	200
310	867	205
320	837	222
330	805	236
340	768	251
350	728	272
360	677	301
370	600	357
375	470	470

agreement was found to be surprisingly good. The other correlating equations were used to give the remaining entries together with some graphical interpolation along the 350°C isotherm. The region covered by the equations is as shown in Fig. 9.

CONCLUSIONS

The equations (1)–(7) presented here provide values of viscosity for compressed water and steam up to a pressure of 1000 bar and temperature of 1000°C. The values are within the tolerances of the International Skeleton Table up to its pressure and temperature limits of 800 bar and 700°C.

It is clear that further experimental work is needed to give a complete definition of the viscosity of water and steam. This is particularly true in the temperature range 0–400°C where further systematic experimentation at all levels of pressure would probably remove existing discrepancies and enable liquid water measurements (< 375°C) to be reconciled with the high pressure steam measurements (> 375°C). It is believed that the values for compressed water, given by equation (1), merge better with the high pressure steam values given by equation (6) than do the corresponding equations appended to the International Skeleton Table. However, as more experimental work comes forward improved correlations become possible and the obscurities in both the compressed water and superheated steam regions should disappear. The authors hope that a sufficient body of data on compressed water, in particular, will accrue over the next two or three years by which time a complete revision should be possible. Unless greater accuracy is shown to be needed there would appear to be no advantage to be gained from further experimental work at advanced temperatures and pressures unless it were extended beyond 1000°C and 1000 bar.

ACKNOWLEDGEMENTS

The work described here forms part of a programme of research on the properties of steam, sponsored by the Central Electricity Generating Board, carried out in the University of Glasgow under the general direction of Professor James Small. The authors wish to thank a number of their colleagues for assistance with computations and the preparation of diagrams.

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Résumé—De nouvelles corrélations et de nouveaux tableaux pour la viscosite de l'eau et de la vapeur d'eau sont présentés. Les corrélations ont le même format genéral que celles jointes aux Tableaux Simplifiés Internationaux, (800 bars, 700°C), mais tiennent compte des travaux récents, permettant ainsi d'obtenir des valeurs allant jusqu'aux limites supérieures de 1000 bars et de 1000°C. L'article donne les références de tous les travaux importants et recommande que des mesures étendues et systématiques soient faites dans la gamme de températures 0–400°C pour toutes les pressions.

Zusammenfassung—Neue Beziehungen und Tabellen für die Viskosität von Wasser und Wasserdampf werden angegeben. Die Beziehungen sind von der gleichen allgemeinen Art wie sie der internationalen Rahmentafel beigefügt sind (800 bar, 700°C), berücksichtigen aber neuere Arbeiten und gestatten damit Werte bis zu 1000 bar und 1000°C abzuleiten. Es werden Hinweise auf alle führenden Arbeiten gegeben und systematische und ausgedehnte Messungen im Temperaturbereich von 0–400°C bei allen Drücken empfohlen.

Аннотация—Представлены новые соотношения и таблицы для коэффициента вязкости воды и водяного пара. Они имеют ту же общую форму, что и соотношения, включенные в Международные Скелетные Таблицы (800 бар, 700°C), но учитывают последние работы в данной области. Это позволяет продвинуться в область более высоких параметров до 1000 бар и 1000°C. В статье приводится обзор всех осноных работ и указывается на необходимость проведения систематических и тщательных измерений в диапазоне температур 0–400°C при всех давлениях.